Channel doping-dependent analytical model for symmetric double gate metal-oxide-semiconductor field-effect transistor. II. Continuous drain current model from subthreshold to inversion region

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(Received 1 April 2013; accepted 17 May 2013; published online 5 June 2013)

Based on the subthreshold region model described in Paper I [Cho et al., J. Appl. Phys. 113, 214506 (2013)], a continuous drain current model with a variation of channel doping concentration (NA) for symmetric double gate metal-oxide-semiconductor field-effect transistor is presented. Here, the inversion region drain current model is derived by solving the long-channel 1D Poisson’s equation due to the strong screening effects by electrons. The continuous drain current model is obtained by interpolating the subthreshold region model and the inversion region model. Since the subthreshold region model includes the short-channel effects, it is shown that the continuous drain current modeling results are in good agreement with commercially available 2D numerical simulation results from the subthreshold to the inversion region in the wide range of NA. © 2013 AIP Publishing LLC [http://dx.doi.org/10.1063/1.4808453]

I. INTRODUCTION

As mentioned in the introduction of Paper I, the multi-gate metal-oxide-semiconductor field-effect transistor (MOSFET) such as double gate (DG) MOSFET has been a candidate to extend the limitation of scaling capability of MOSFET. Due to the growing interest in DG MOSFET, there have been a lot of proposed analytic drain current models for characterizing DG MOSFET. Taur et al., Lu et al., and Oritz-Conde et al. proposed DG MOSFET analytic model. However, these models were only applicable in undoped or lightly doped long-channel device. In order to overcome the limitations, attempts have been made to model the drain current for short-channel DG MOSFET. However, these models only derived subthreshold current (IDSSub) and failed to express inversion region drain current (IDSInv). Therefore, a continuous drain current model from subthreshold to inversion region is required to fully describe the characteristics of DG MOSFET. Previously, Tsormpatzoglou et al. and Kolberg et al. reported a continuous drain current model of lightly doped DG MOSFET. There were only few papers proposing a continuous drain current model covering a wide range of channel doping concentration (NA).

In this paper, a continuous drain current model is proposed by using interpolation method of the subthreshold region model (i.e., derived in Paper I) and the inversion region model with respect to NA. By introducing the subthreshold region model, the analytical model includes the short-channel effects such as subthreshold swing (SUSS) degradation and threshold voltage (VTH) roll-off. Finally, the proposed model is compared with the commercially available 2D ATLAS numerical simulation for validation.

II. ANALYTICAL MODEL

The schematic structure of a symmetric DG MOSFET used in our analysis is shown in Fig. 1, where L is the channel length, tsi is the channel film thickness, and tox is the gate oxide thickness. In addition, the points at x = 0 and y = 0 indicate the source-channel interface and the center of the channel, respectively. A uniform p-type doping concentration, NA, is assumed in the channel region. The source and drain regions are also assumed to be heavily doped with n-type doping concentration ND = 10^20 cm^-3 in this work.

A. Subthreshold region model

We briefly summarize the derivation of subthreshold region drain current as described in Paper I. In the subthreshold region, the mobile carriers in the channel can be neglected. The channel potential ψ(x,y) can be obtained by solving the 2D Poisson’s equation

\[ \frac{\partial^2 \psi(x,y)}{\partial x^2} + \frac{\partial^2 \psi(x,y)}{\partial y^2} = \frac{qN_A}{\epsilon_{si}}, \]

where \( \epsilon_{si} \) is the silicon permittivity.

FIG. 1. Schematic structure of a symmetric DG MOSFET where L, tsi, and tox are the channel length, channel film thickness, and gate oxide thickness, respectively.
Using the evanescent method, the full 2D channel potential is obtained as

$$\psi(x, y) = \frac{qzA}{\varepsilon_{st}} y^2 + V_{GS} - V_{fb} - \frac{qN_{A}t_{ox}}{2\varepsilon_{ox}} - \frac{qN_{A}t_{si}}{8\varepsilon_{si}}$$

$$+ \cos \left( \frac{y}{\lambda_{t}} \right) \times \left( \frac{U_{1}\sinh \frac{x}{\lambda_{t}} + U_{1}\sinh \frac{L - x}{\lambda_{t}}}{\sinh \frac{L}{\lambda_{t}}} \right), \quad (2)$$

where $\varepsilon_{ox}$ is the SiO$_2$ permittivity, $V_{GS}$ is the gate-source voltage, and $V_{fb}$ is the flat band voltage, while $\lambda_{t}$, $U_{1}$, and $V_{t}$ are the constants determined from the boundary conditions.

By adapting the concept of the virtual cathode, we can extract the $x_{\text{min}}$ where the minimum potential is located as

$$x_{\text{min}} = \frac{\lambda_{t}}{2} \ln \left[ \frac{V_{1}\exp \left( \frac{2L}{\lambda_{t}} \right) - U_{1}\exp \left( \frac{L}{\lambda_{t}} \right)}{V_{1}\exp \left( \frac{L}{\lambda_{t}} \right) - V_{1}} \right] \quad (3)$$

Assuming that $I_{DS_{sub}}$ is mainly dominated by the diffusion in the subthreshold region, $I_{DS_{sub}}$ can be finally derived as the sum of front-gate component subthreshold current ($I_{f_{sub}}$) and back-gate component subthreshold current ($I_{b_{sub}}$). $I_{f_{sub}}$ and $I_{b_{sub}}$ are described as

$$I_{f_{sub}} = \frac{W}{L_{e}} q\mu_{n} V_{t} \frac{n_{i}^{2}}{N_{A}} \left[ 1 - \exp \left( - \frac{V_{DS}}{V_{t}} \right) \right] \frac{V_{t}}{E_{f}} \left[ \exp \left( \frac{\psi_{\text{min}}(-\frac{L}{2})}{V_{t}} \right) - \exp \left( \frac{\psi_{\text{min}}(0)}{V_{t}} \right) \right], \quad (4)$$

$$I_{b_{sub}} = \frac{W}{L_{e}} q\mu_{n} V_{t} \frac{n_{i}^{2}}{N_{A}} \left[ 1 - \exp \left( - \frac{V_{DS}}{V_{t}} \right) \right] \frac{V_{t}}{E_{b}} \left[ \exp \left( \frac{\psi_{\text{min}}(0)}{V_{t}} \right) - \exp \left( \frac{\psi_{\text{min}}(\frac{L}{2})}{V_{t}} \right) \right], \quad (5)$$

where $W$, $L_{e}$, $\mu_{n}$, $V_{t}$, $n_{i}$, and $V_{DS}$ are the channel width, effective channel length, electron mobility, thermal voltage (i.e., $V_{t} = kT/q$), intrinsic carrier concentration, and drain-source voltage, respectively. $E_{f}$ and $E_{b}$ are described as

$$E_{f} = \left( \psi_{\text{min}}(-\frac{L}{2}) - \psi_{\text{min}}(0) \right), \quad (6)$$

$$E_{b} = \left( \psi_{\text{min}}(\frac{L}{2}) - \psi_{\text{min}}(0) \right). \quad (7)$$

**B. Inversion region model**

It is noted that the inversion region drain current can be obtained from the long channel approximation of the channel potential due to the strong screening effects by electrons. In the inversion region, the mobile carriers in the channel cannot be neglected anymore. Due to the long channel approximation, we can solve the modified 1D Poisson’s equation described as

$$\frac{d^{2}\psi_{\text{inv}}(y)}{dy^{2}} = \frac{q}{\varepsilon_{si}} \left( n + N_{A} \right) = \frac{q}{\varepsilon_{si}} \left( \frac{n_{i}^{2}}{N_{A}} \exp \left( \frac{\psi_{\text{inv}}(y) - V}{V_{t}} \right) + N_{A} \right), \quad (8)$$

with the following boundary conditions:

$$\varepsilon_{ox} \left[ V_{GS} - V_{fb} - \psi_{\text{inv}} \left( \pm \frac{L_{e}}{2} \right) \right] = \pm \varepsilon_{si} \left| \frac{\partial \psi_{\text{inv}}(y)}{\partial y} \right|_{y = \pm \frac{L_{e}}{2}}, \quad (9)$$

where $\psi_{\text{inv}}(y)$ is the long channel approximation of the channel potential in the inversion region and $V$ is the electron quasi-Fermi potential. It can be noted that the $V$ is constant in the y-direction. Due to the complexity of solving Eq. (8) directly, we adapt the parabolic channel potential approximation method proposed by Young which is expressed as

$$\psi_{\text{inv}}(y) = A + By + Cy^{2}, \quad (11)$$

where $A$, $B$, and $C$ are the arbitrary constants to be determined from the boundary conditions.

Substituting Eq. (11) back into Eq. (8) and solving with the help of Eqs. (9) and (10), we obtain the following expressions:

$$A = V_{GS} - V_{fb} - \left( \frac{L_{e}^{2}}{4} + \frac{L_{e} \mu_{n} V_{t}}{2\varepsilon_{ox}} \right) C, \quad (12)$$

$$B = 0, \quad (13)$$

$$2C = \frac{q}{\varepsilon_{si}} \left( \frac{n_{i}^{2}}{N_{A}} \exp \left( \frac{A + Cy^{2} - V}{V_{t}} \right) + N_{A} \right). \quad (14)$$

Putting Eq. (12) to Eq. (14) for $y = L_{e}/2$, we can determine the values of arbitrary constants (i.e., $A$ and $C$) by the following expression:

$$2C = \frac{q}{\varepsilon_{si}} \left( \frac{n_{i}^{2}}{N_{A}} \exp \left( \frac{V_{GS} - V_{fb} - \frac{L_{e} \mu_{n} V_{t}}{2\varepsilon_{ox}} C - V}{V_{t}} \right) + N_{A} \right). \quad (15)$$
For a given $V_{GS}$ and $N_A$, $C$ can be solved as a function of $V$. From the source to drain, the $V$ varies from the 0 to $V_{DS}$. By using the similar method performed by Taur et al., the inversion region drain current is calculated from Pao-Sah’s integral:

$$
I_{DSinv} = \frac{W}{L_c} \mu_n \int_0^{V_{DS}} Q(V) dV = \frac{W}{L_c} \mu_n \int C_0 \frac{dV}{dC} dC,
$$

where $Q$, $C_S$, and $C_D$ are the total charge density, solutions to Eq. (15) when $V = 0$ and $V = V_{DS}$, respectively. From the Gauss’s Law,

$$
Q = \int_{si}^{t_{ox}} \varepsilon_s dy/C_0,
$$

By differentiating Eq. (15) with respect to $C$, we can obtain the following expression:

$$
\frac{dV}{dC} = \frac{\varepsilon_o}{\varepsilon_s} \frac{qN_A}{2\varepsilon_s} \left( \ln \left( \frac{C_S}{C_D} \right) - \ln \left( \frac{qN_A}{2\varepsilon_s} \right) \right). 
$$

$\frac{dV}{dC}$ is obtained by solving Eq. (16) with the help of Eqs. (17) and (18) described as

$$
I_{DSinv} = \frac{W}{L_c} \mu_n \int_0^{V_{DS}} Q(V) dV = \frac{W}{L_c} \mu_n \int C_0 \frac{dV}{dC} dC.
$$

Fig. 2 shows the graph of $I_{DSinv}$ vs. $V_{GS}$ for $V_{DS} = 0.1$ V. It is shown that the model can be divided into two regions, $N_A$ dominant region (i.e., depletion charges) and $n$ dominant region (i.e., mobile carriers). As can be seen from Fig. 2, $I_{DSinv}$ begins to increase when the number of the mobile carriers exceeds the number of the depletion charges.

### III. MODEL VERIFICATION AND DISCUSSION

We derived the $I_{DSsub}$ and $I_{DSinv}$ model in Sec. II. However, these models are not continuous itself. In order to obtain the continuous drain current model from the subthreshold to the inversion region, a suitable interpolation function that matches $I_{DSsub}$ and $I_{DSinv}$ near the threshold voltage is required. The interpolation function used in our model has the following form:

$$
I_{DS} = \frac{I_{DSsub} \times I_{DSinv}}{(I_{DSsub}^m + I_{DSinv}^m)^{1/m}}
$$

where $I_{DS}$ is the continuous drain current model and $m$ is the parameter that determines the shape of $I_{DS}$ in the knee region.

Fig. 3 shows the schematic illustration of the interpolation method. It is shown that $I_{DS}$ well follows $I_{DSsub}$ in the subthreshold region and $I_{DSinv}$ in the inversion region.

In order to validate our proposed model, a commercial 2D ATLAS numerical device simulator provided by Silvaco, Inc. is used. Here, the constant electron mobility ($\mu_n = 1417$ cm$^2$/Vs) is used in both the analytical model and 2D numerical simulation. It can be noted that the $V_{fb}$ in our model is assumed to be the same as the ATLAS simulation model.
Figs. 4(a) and 4(b) show the comparison of $I_{DS}$-$V_{GS}$ characteristics in semi-logarithmic and linear plots between our analytical model and the simulation results for $L$-dependence and $N_A$-dependence, respectively. The analytical model results and simulation results are represented as solid lines and open symbols, respectively.

As shown in Fig. 4(a), our analytical modeling results are well matched with the simulated results in all operation regions (i.e., from subthreshold to inversion region) for the wide range of $L$. Due to introducing the subthreshold region model, our analytical model well expresses the short-channel effects such as $S_{sub}$ degradation and $V_{TH}$ roll-off that the long-channel model cannot express itself. In Fig. 4(b), it is also shown that our analytical model results are well matched with the simulated results with the variation of $N_A$. It is shown that $N_A$ has low effect on $V_{TH}$ when $N_A$ is less than $10^{17}$ cm$^{-3}$ which is also verified in Paper I.

The analytical model has been also validated by comparing transconductance ($g_m$) with the simulated results. The comparison of $g_m$ characteristics between the analytical model and simulation results is shown in Figs. 5(a) and 5(b) for $L$-dependence and $N_A$-dependence, respectively.

It is shown that our analytical model is well matched with the simulation results. However, the slight deviation between the analytical model results and simulation results occurs in the transition region (i.e., from the subthreshold to the inversion region). The error can be occurred from the subthreshold region model due to the omission of the higher order terms as mentioned in Paper I.

**IV. CONCLUSION**

Starting from the concept that the inversion region drain current was obtained from the long channel approximation, we derived the inversion region drain current model with the variation of $N_A$ by solving the 1D Poisson’s equation. We built a continuous drain current model with the subthreshold region model (derived in Paper I) and the inversion region model by using the interpolation method. The subthreshold region model expressed the short-channel effects which the inversion region model cannot represent. The results were compared with commercially available 2D numerical simulation. It was observed that the analytical model well represented the current-voltage characteristics from subthreshold to inversion region in the wide range of $N_A$. Therefore, the continuous drain current model can be applicable to the compact model of DG MOSFET.

**ACKNOWLEDGMENTS**

This work was supported by the IT R&D program of MKE/KEIT [10039174, Technology Development of 22 nm level Foundry Devices and PDK].
